Ms4   AN EXPLORATION OF THE BEGINNINGS OF METROLOGY

THE NUMBER 7 AND IRRATIONAL NUMBERS √2, √3 & PI

SYNOPSIS

The Sumerians of the Fertile Crescent, Mesopotamia are credited with the first use of mathematics. They developed a metrological system, first decimal based and then sexagesimals, 60 based, bequeathing to us the 60 seconds/60 minutes of an hour; 360 degrees in a circle and 3 x 60 or 180 degrees in a triangle.

But from the earliest periods of history in Europe, c35000 – 20000BCE, there is evidence of counting systems. Thus by 4000BCE megalithic metrology may have been the norm for our ancestors.

THE PALAEOLITHIC PERIODS   Diagrams Ms4D01, Ms4D02

Intimations of counting and numeracy in this period come from the many notched bone and stone pieces found by archaeologists. These can be accurately dated. They include a notched bone of the Magdalenian period, 19000-12000BCE, found at Brassempouy, Landes, France, with notches in groups of 3, 5, 7 & 9. This enables counting, the mathematics of addition and subtraction to be carried out.

At La Grotte de Remonchamps, Belgium, a small bone with a series of groups of five holes was found in 1902. Then a further excavation in 1970 uncovered another. Both were dated to +10000BCE and were practically the same length at 70 cms.

Analysis of these two bones led to the belief that a large and progressive counting system could easily be achieved.

But most surprising of all is the discovery made in 1969 at Bilzingleben, (east) Germany. The site variously dated to 250000-300000BCE provided the remains of four bones. Not just any bones but Elephant Bones, deliberately carved with marks of similar depth and made by a single tool in a single operation. The number of notches, a three sequence carving is, 7: 14: 7.

One author in 1998 commented on these bones and the quinary numerical system, stating that, *"there is no evidence to suggest that the peoples of the Upper Palaeolithic period were in any way the intellectual inferiors of modern humans, as we have seen, the counting systems used in tribal cultures down to recent times may also be rather complex"*

In a chapter (6) entitled 'Palaeoscience' that same author commences as follows; *"One fundamental and recurring theme of this book is the idea that in seeking to understand the human story it is not possible to do so with any degree of accuracy without taking into account the innovations and developments that occurred in the Stone Age, and I shall now turn my attention to the primordial origins of mathematics and the roots of science."*

The index to this book under Mathematics has, "Mathematics and Numerical Systems", "Engravings" and "Significant numbers (7 & 14), as numerated headings.

CONTINUITY OF IDEAS AND CONSTRUCTIONS; EUROPE

It is necessary to search for the oldest evidence of man's intellectual activities and these will be found in the evidence of the oldest recorded actions. That is the erection of Stones, Stone Circles and similar landscape phenomenon. The power to perceive and imagine is not intellectual activity. Ancient man must have taken actions, produced objects or systems which interact with one another.

"By definition a megalith is not a chance phenomenon, but was erected through deliberate human effort. WHY? HOW? Moving such massive objects obviously required a sound understanding of traction, elevation, material resistance, in short an extensive fount of knowledge. The problem lies in assessing how far prehistoric "engineers" and "worshippers" progressed in theoretical understanding".

We are all fully aware of the hundreds of Standing Stones and Stone Circles in the British landscape, but this activity is in fact spread far and wide. It is a Europe and World Wide phenomenon. They exist in Scandinavia, France, Ireland, Eastern Europe, Arabia, N.Africa, Palestine (as was), Sinai, Assam and India.

These are the ancient migration routes of man, the agrarian and science distributors.

WRITING, WORDS AND NUMBERS   Diagram Ms4D03

The earliest known writing appeared pre 3200BCE in and around the land of Sumer, between the Tigris and Euphrates rivers. Around 3200BCE they also devised a written numerical notation system, part of which is the sexagesimal, 60 based system. It had previously been an oral system and used the decimal base 10 as an intermediary.

However counting systems had been around since at least the 9th millennium BCE, in a Mesolithic and Neolithic Middle East, evinced by the thousands and thousands of 'Calculi', which are small objects in a wide variety of sizes and regular geometric shapes. They have been determined by archaeologists as tokens, counting tokens.

Following the Sumerians, a Babylonian text dated 1800-1700BCE has the relationship of the three sides of a right angled triangle determined as; $A^2 = B^2 + C^2$, known to us as the "Pythagorean Theorem". But his theorem was only established in c530BCE.

The earliest known documented record of the PI ratio was written on what is now known as the Rhind Papyrus. This paper implies that the ratio of Circumference to Diameter equals 256/81; i.e. and thus we know, $\prod D$ = Circumference, and $\prod R^2$ = area contained in and of a circle.

(Curiously, Claudius Ptolemy used a circle divided into 83 parts in his calculations.)

At the same period the Babylonians were using 25/8 for Pi, and then in the 3rd C. BCE, Archimedes used a 96 sided polygon to establish PI as between $3\ {}^{10}/_{71}$ to $3\ {}^{1}/_{7}$, i.e. ${}^{223}/_{71}$ to ${}^{22}/_{7}$.

Claudius Ptolemy in the 2nd C. CE uses, PI = $3^0\ 8'\ 30"$ or ${}^{377}/_{120}$.

When we investigate $\sqrt{2}$, another irrational number which can only be approximated (quite closely though) by a ratio of integers, which can be found quite easily by geometry. A straight edge and compass will enable you to draw a square whose diagonal is $x\sqrt{2}$ larger than one of the four sides.

A Mesopotamian tablet shows a square with its diagonals: on one side is written '30', under one diagonal 42, 25, 35 and along the same diagonal above is 1, 24, 51, 19. (i.e. $1 + {}^{24}/_{60} + {}^{51}/_{60x60} + {}^{10}/_{60x60x60}$)

This third number is the correct value of $\sqrt{2}$ to four sexagesimal places, which is equivalent to our decimal 1.414213; virtually correct. The second number is the product of the third and first and so gives the length of the diagonal when the side is '30'. Note. The original clay tablet is in the Yale University Museum, USA.

The Sumerians gave us the 360 degrees of a circle, and thus the 180 degrees of a triangle. After the 45/45/90 degree or $1:1:\sqrt{2}$ triangle, the next is the 30/60/90 degree or $1:\sqrt{3}:2$ triangle.

The fact that the numerical written notation system was derived c3200BCE points to a much earlier discovery which necessitated the above facts being written down as the knowledge increased in complexity.

MEGALITHIC CONSTRUCTION TIMESCALES   Diagram Ms4D04

The simplest method to illustrate such a timescale is by a chart diagram. There-on are the pertinent dates here-in discussed, from the Middle East to Britannia.

It illustrates that there was a possible time frame for the transference of data from the Sumerian/Babylonian heartlands via Anatolia and the Balkans to Europe. This is also the probable migration route for peoples.

When the first farmers encouraged planting and there was a surplus or at least there was no necessity to remain peripatetic hunter-gatherers, the population exploded exponentially. This is evinced by the excavated towns and cities of the Middle East and Anatolia, and even into Romania. As mentioned in Ms3, the Anatolian City of Çatal Hüyük had a population of some 7000 persons and was founded c6000BCE. They constructed a multi-storey city, 14 levels, which required great skill and knowledge.

In Romania, the Cucuteni Chalcolithic civilisation is dated in stages ranging from 4500BCE to 3500BCE. Over 300 settlements were discovered in the region, with one near Tal'ianki in the rivers Bug/Dneiper area, covered a surface area of 450 hectares and had some 2700 constructions arranged in two concentric circles. Compare this to the Egyptian development and we see rather unknown civilisations eclipsing all.

Thus it is fair to opine that the way was open for transference of ideas and metrological information across southern Europe to Western Europe.

MEGALITHIC MEASUREMENT

We can only assess the likelihood of a measurement being used from the structures that may include the necessity to measure. In paper Ms3 at Sarn Y Bryn Caled in Wales, it was shown that a single measurement was at the heart of the Timber Post Circles design.

That measurement was 33/7 statute feet, 4.714 feet, 56.5714 inches and that it was linked to the Megalithic Yard, 2.722 feet and the Megalithic Fathom, 5.444 feet determined by Professor Alexander Thom from his survey work on the megalithic sites.

I have named the 33/7 unit the Unit Yard, which is a quantum found in Britain and Scandinavia. But can it be shown to have an ancestry in the Middle East or there-about.

SEARCHING FOR ANTECEDANTS

The information concerning ancient measurements was collated in the 19th century by scholars practising Cambistry, the science of exchange, weights and measures &c.

The measures of the Middle East were generally noted by comparison to each other, particularly to well known measures such as the finger (doigt) and cubit (arm). One of the first (noted alphabetically) is the Acène, Acena, Décapode;

"Mesure de longueur des anciens, formant la 10e partie du Plèthre. C'etait leur perche d'arpentage. En Egypte, L'acène philétérienne vailat $1^2/_3$ Orgyce philétérienne = 5 coudées de 2 pieds = $6^2/_3$ coudées philétériennes = 10 pieds philét. = 40 palmes ou palestes = 160 dactyles = **3.937 statute yards.**

A simple investigation of this measure provides a clue to the origins of megalithic measures. Acène = 3.937 yards or 11.811 feet or 141.732 inches. Thus each dactyle or finger is 141.732 ÷ 160 which equals 0.885825 inches or 22.5mm per unit.

Divide the Acène of 11.811 feet by 4.714 feet or the inch measures 141.732 ÷ 56.5714 and we have 2.5054 units. Therefore we can safely assume the Acène was 2.5 unit yards.

But if we divide 160 by 2.5 the resultant is 64, and the finger or dactyle becomes 56.5714 ÷ 64, which equals 0.88393 compared to 0.885825. This is a difference of 20 thousandths of an inch!

GEOMETRY   Ms4D05, Ms4D06

In Babylonia the irrational number √2 was determined and the proof is the clay tablet. However it was calculated using sexagesimals and when translated into a fraction or ratio number becomes 99/70. The comparison is 1.414285714 to 1.414213562, and as such 99/70 is a highly accurate substitute being the square root of 2.0002041.

It is easily confirmed by geometry. Draw a 45 degree equilateral triangle with side lengths 70 units. Project the hypotenuse line to the extended base line and the answer can be read, 99 units with a virtually impossible error readable.

But, extend that triangle and the next irrational number appears √3, decimally 1.73205081.

Further extensions of the triangle indicate and then allow the other irrational numbers to be evaluated.

But, if we commence with the 45 degree triangle with side length 40 units, the √2 figure becomes 56.5685 units, which divided by 12 produces a unit of 4.714 or 33/7 units.

If we now draw a triangle having 30/60/90 degree angles and use the plain 56 as the base measure, the sides are in the ratio 1:√3:2 and the √3 side is measured as 97 units and thus it is possible to use the ratio of 97:56 as a representation of √3.

The Sumerians/Babylonians knew the "Pythagorean theorem" and thus they could interpolate the 1:√3:2 ratio triangle to find √3 as follows;

√ [(112 x 112) – (56 x 56)] = √3. That is, 12544 – 3136 = 9408, the calculated square root of which is 96.9948, and the original 97 x 97 equals 9409.

This would have been an infinitesimal difference when a triangle was drawn on parchment or similar. But, in trying to increase the accuracy, draw the triangle as 56.5/113 units and √3 becomes 97.861 units, or decimally 1.732053097, still remarkably accurate. However, increase the unit to 98/56.5 and the figure is 1.7345 which is increasingly in-accurate.

In the mathematical mind there would have been the thought that irrational numbers may have a common base, and the number 7 may provide an answer. It is plausible to consider that √3 may well have been 343/198 which is quite clearly a 7 base number.

Fractions equal; $\dfrac{7 \times 7 \times 7 \times 7}{7 \times 20 \times \sqrt{2}}$ or $\dfrac{7 \times 7 \times 7 \times 70}{20 \times 99}$ and thus as $\sqrt{2} = 99/70$ and can be written as 3/10 x 33/7. There is the direct link mathematically to the unit yard ratio.

Decimally it is 1.732323232 which when squared equals 3.000944, and is quite therefore accurate.

Thus Megalithic measures, the Yard, Fathom and Unit Yard which have their relationship determined by the $1:\sqrt{3}:2$ ratio triangle may also arise from a far simpler metrology the human body. Did Mesolithic or Neolithic man originate measures by using the most basic of all instruments, the human hand?

In **Ms3**, it was stated that this authors hand was c120mm and that the twelfth part of the Megalithic Unit Yard was 119.75mm. That hands index finger is c20mm and the sixth part of 119.75 equals 19.958mm. Thus the Unit Yard may have had 72 base units.

But, as the Sumerians/Babylonians used the "Pythagorean Theorem", the resolution of the $1:\sqrt{3}:2$ ratio triangle, in basic units would be as follows;

If the $\sqrt{3}$ unit is 33/7, then 1 = 49/18 and 2 = 49/9,

And thus 49/9 x 49/9 = (49/18 x 49/18) + (33/7 x 33/7).

The proof is as follows; substitute X for 33/7;

Therefore, $X^2 = (98/18 \times 98/18) - (49/18 \times 49/18)$ or $X^2 = {}^{7203}/_{324}$. Thus $X^2 = 200/9$ (i.e. ${}^{7200}/_{324}$)

But, $X^2 = 200/9$ or therefore $X^2 = (10\sqrt{2} \div 3) \times (10\sqrt{2} \div 3)$, and thus $X = 10\sqrt{2} \div 3$.

Finally $\sqrt{2} = 99/70$ and therefore we have $(10 \times 99) \div (3 \times 70)$ or 33/7.

Substitute a measurement for the unit number and there is a coherent system for megalithic man to have used, the Unit Yard.

IRRATIONAL NUMBERS COMBINE

Consider this. What has just been shown is that the irrational numbers PI, $\sqrt{2}$ and $\sqrt{3}$, as well as a ratio measure used in metrology all have the same base.

Pi = 22/7: $\sqrt{2} = 99/70$: $\sqrt{3} = 343/198$ and the Unit yard is a ratio 33/7 units.

They are all capable of being expressed as a factor of each other. They are therefore inter-changeable in a mathematical equation, and without a pocket calculator handy, substitution is the only practical method of solving some equations. If you are inventing a system of mathematics, that would surely be a pre-requisite.

CONCLUSION

Mathematics and measurement were developed in the Middle East, basically Mesopotamia and spread both Westerly and Easterly across the continents. There they developed a highly sophisticated sexagesimal system from an original decimal additive system. They developed answers to the irrational numbers in the form of fractions or ratios.

There was then a trend westwards by an expanding populace who met other sophisticated civilisations and gradually the mathematical as well as the agrarian principles were transmitted and thus transported across the continents.

The less sophisticated, although equally intelligent peoples of the far west of Europe learnt the basic geometry and applied it to their monuments in the form of shape and measure.

Thus we find in the landscape of Western Europe a quantum measure which is part of the Middle East metrology.

<div align="right">Michael J Ferrar</div>

ARCHAEOLOGISTS CAUTION

I have not actively sought out a measurement attributable to Megalithic man. My reading matter over many years has been quite eclectic and I have gradually evaluated the data there-in and then found that this research was so very necessary.

However, when I started seeking actual examples of ancient mans mathematical and geometric knowledge the following text was a cautionary tale, but also a spur to do better.

In his book, "Science and Society in Prehistoric Britain", Dr E Mackie wrote the following;

"An elementary warning should be written with letters of fire on the mind of every archaeologist who makes use of mathematical and statistical techniques, or conclusions drawn with their aid by

someone else. M H Moroney puts it clearly, 'it is true that it is extremely difficult to interpret figures when they relate to some concrete problem. It is equally true that it is extremely easy to do arithmetic. Here-in lies the real difficulty. Averages can be calculated to nineteen places of decimals with astonishing ease. When the job is done it looks very accurate. It is an easy and fatal step to think that the accuracy of our arithmetic is equivalent to the accuracy of our knowledge about the problem in hand. We suffer from delusions of accuracy. Once an enthusiast gets this disease, he and all who depend on his conclusions for their welfare are damned.'

That caution is only realistic if you do not acknowledge the original methods that would have been utilised by ancient man and endeavour to work in a similar manner. Thus everything, every number must be a fraction, a ratio and the calculator should be banned. Given that Archaeologists did not recognize that measurement was part of their remit it is no surprise that they did not recognize when to use mathematics.

<div align="right">Michael J Ferrar</div>

APPENDICES; GENERAL NOTES

1) Armenia and Asia Minor in antiquity.
   La mille etait 7 grandes asparez = 10 asparez ordinaire = 1000 pas = 6000 pieds phileteriens
   Asparez, 7^e part du mille, etait egal a $1^3/_7$ asparez ordinaire
   $142^6/_7$ pas = $857^1/_7$ pieds phileteriens = 337.462 statute yards.

2) The Austrian Miele (mile)
   The Miele is 4.714 statute miles. That is 4.714 inches x 63360; 4.714 feet x 5280 and 4.714 x 1760 yards.
   The Austrian system is in fact now, 288000 Zoll, 24000 Fuss and 4000 Klafter equal the Miele. But there is also the remnant of earlier measures in the 33 inch unit that exists. And that unit is of course 7 x 4.714 inches or 7 x 33/7 inches.

3) The Basque League and Terrestrial Geometry
   Papers published in Archaeoastronomy discuss the origins of the Iberian Vara unit and its utilisation in the construction of Stone circles in the Basque country and northern Spain.
   The base 7 mathematical system and the Vara standard used by the Basques in laying out circles also appear to have been employed by them to determine the circumference of the earth.

4) Cambridge Ancient History-III. The Sumerian Revival
   "Emutbal, a late (Elamite?) name for one of the oldest Sumerian halting places, was designated by the Sumerian ideogram for 7, a mystic number given also to Erech and the sacred city of Kish in Sumer"

5) The number 7 by James Hadley LL.D
   "It is well known that men of different times and nations have associated with particular numbers the idea of a peculiar significance and value. It is also well known that, of all numbers, there is one which exercised in this way a wider influence, no one which has commanded in a higher degree the esteem and reverence of mankind, than the number 7. The mystic pre-eminence of this sacred number is as ancient as it is venerable."
   (Rather full of hyperbole but never-the-less it sums up the situation apropos the number 7.)

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He was Professor and Chair of Engineering Science, Oxford University.

9) THOM Archibold S, 1995, 'Walking in all of the Squares', Argyll Publishing.
This book is a tribute to his father, Alexander Thom whom he assisted in many of the surveys and research projects. It contains a wealth of information in the form of essays by many learned academics on the subject of Megaliths, their position in the landscape, their astronomical significance and of course the measurements utilised.